

Consider a Bézier curve. It is given by a polynomial map $B(t) : [0, 1] \rightarrow \mathbb{R}^2$ (where \mathbb{R}^2 means $\mathbb{R} \times \mathbb{R}$, the two-dimensional Cartesian plane). Or, if we take the whole curve, the map is $B(t) : \mathbb{R} \rightarrow \mathbb{R}^2$. Explicitly, the map is

$$B(t) = b_0(t)p_0 + b_1(t)p_1 + b_2(t)p_2 + b_3(t)p_3 \quad (t \in \mathbb{R}),$$

where $b_0(t), b_1(t), b_2(t), b_3(t)$ are the Bernstein polynomials of degree 3 and p_0, p_1, p_2, p_3 are the control points (vectors in \mathbb{R}^2). The plane \mathbb{R}^2 can be identified with the complex plane \mathbb{C} . So, we can view the map as $B(t) : \mathbb{R} \rightarrow \mathbb{C}$. Then it is

$$B(t) = b_0(t)w_0 + b_1(t)w_1 + b_2(t)w_2 + b_3(t)w_3 \quad (t \in \mathbb{R}),$$

where w_0, \dots, w_3 are the complex numbers corresponding to p_0, \dots, p_3 . But this map is a polynomial (each $b_0(t), \dots, b_3(t)$ is a polynomial), therefore it can be extended to a map on the whole \mathbb{C} by extending each $b_0(t), \dots, b_3(t)$ to a map $\mathbb{C} \rightarrow \mathbb{C}$ in the natural way. Then we have the map $B(z) : \mathbb{C} \rightarrow \mathbb{C}$,

$$B(z) = b_0(z)w_0 + b_1(z)w_1 + b_2(z)w_2 + b_3(z)w_3 \quad (z \in \mathbb{C}).$$

This is a polynomial $\mathbb{C} \rightarrow \mathbb{C}$ of degree 3. This means that it is conformal everywhere where the derivative $B'(z)$ does not vanish! See, for example,

<https://mathworld.wolfram.com/ConformalMapping.html>

or

https://www-users.math.umn.edu/~olver/ln_/cml.pdf

(page 35). If the original $B(t)$ is not constant (the curve is not just one point) so that $B'(z)$ does not vanish identically, the condition of non-vanishing of $B'(z)$ excludes at most two points in the plane.

When we identify \mathbb{C} back with \mathbb{R}^2 , we have in our hands a conformal map $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ (conformal almost everywhere).

In this manner any non-constant Bézier curve $B(t)$ produces a conformal map $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ (excluding at most two points). Studying a little further one sees that the image of any straight line segment under this map is an arc of some Bézier curve and can therefore be drawn in Gimp exactly. Consequently, *the image of any figure consisting of straight line segments can be drawn in Gimp exactly, and the mapping is conformal except possibly for at most two points.*