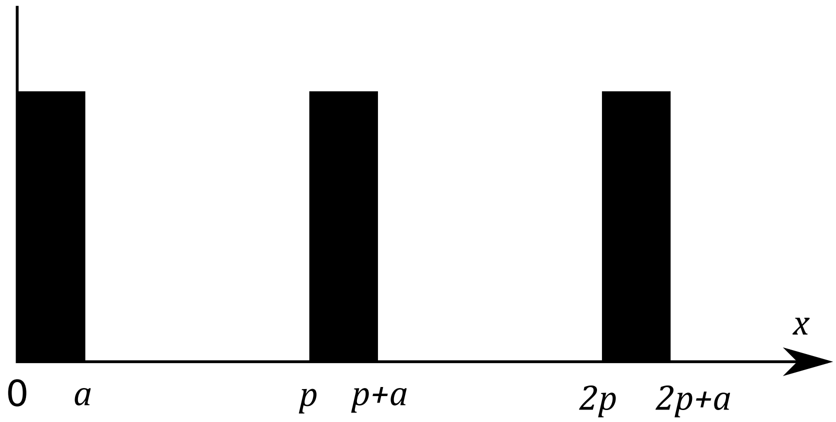
Denote

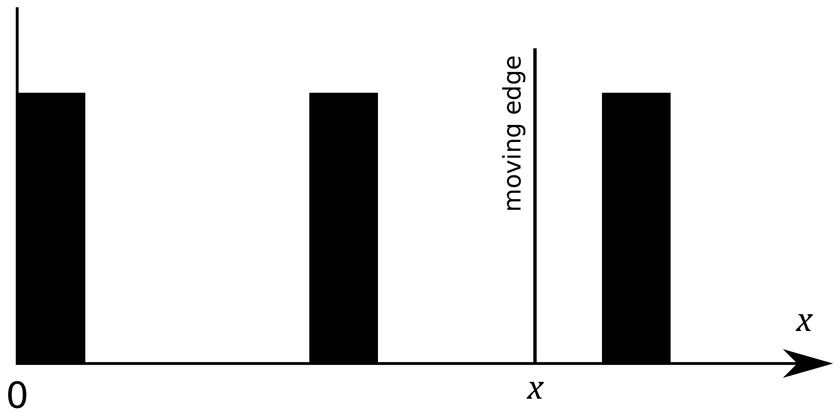
$$\begin{cases} a = \text{lines width}, \\ s = \text{spaces width}, \\ p = a + s \qquad (p \text{ for "period"}), \\ w = \text{window width}. \end{cases} \tag{1}$$

Choose coordinates so that $x = 0$ at the left edge of some fixed line.



Instead of the original problem of a *moving window*, we consider *one moving vertical edge*. We take a vertical edge, and we look at the lines which lie to the left of it, starting at point 0. More precisely, letting the edge be at point x, we shall find the number

$$f(x) = \text{the total width of lines between 0 and } x. \tag{2}$$

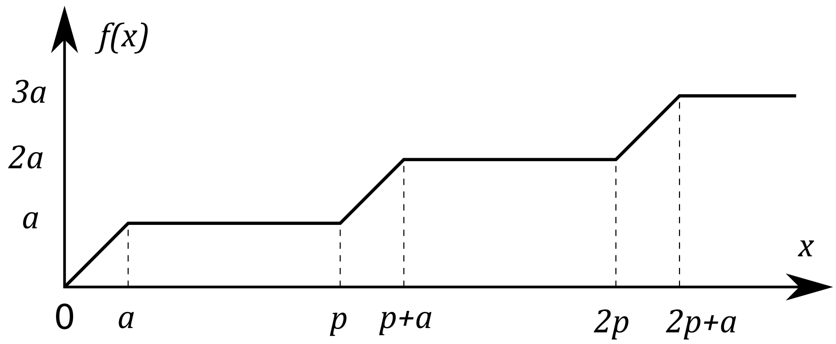


Then the original problem of the moving window will have the answer

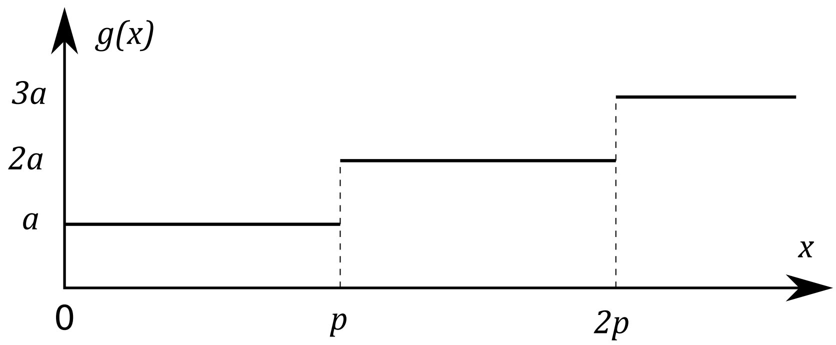
$$\text{the total width of lines inside the window} = f(x + w) - f(x) \tag{3}$$

when the window has its left edge at point x. Namely, the difference of f for the right and left edges of the window tells what is inside the window.

So it suffices to find a formula for the function $f(x)$. It has the following graph:



We start with the following function which we call $g(x)$:



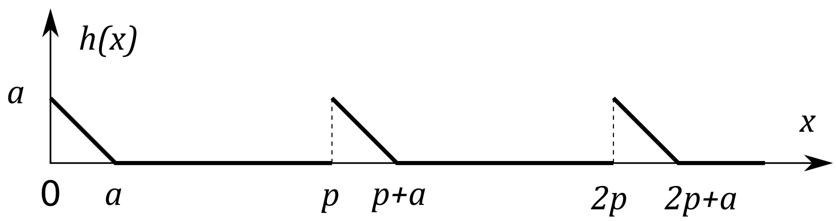
Its formula is

$$g(x) = \left(\left\lfloor \frac{x}{p} \right\rfloor + 1 \right) a \tag{4}$$

where $\lfloor * \rfloor$ is the *floor function.* (Please google for the floor function. Python has the floor function built-in in the math module, available after "import math".) From the pictures it is clear that

$$f(x) = g(x) - h(x) \tag{5}$$

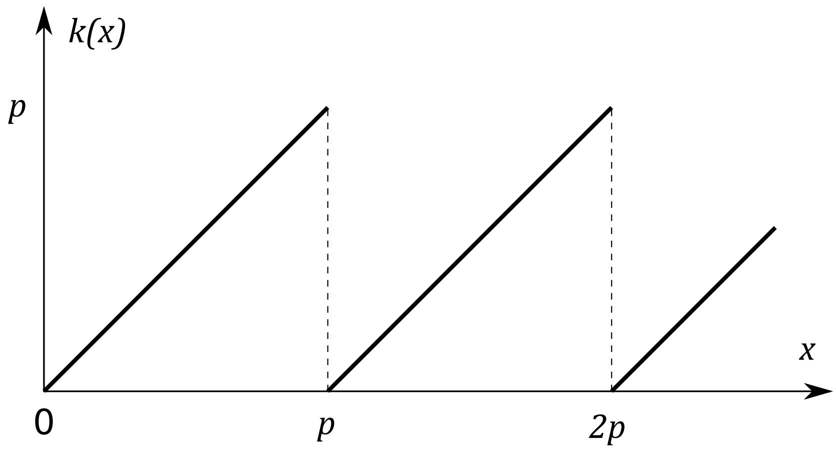
where $h(x)$ is the following function:



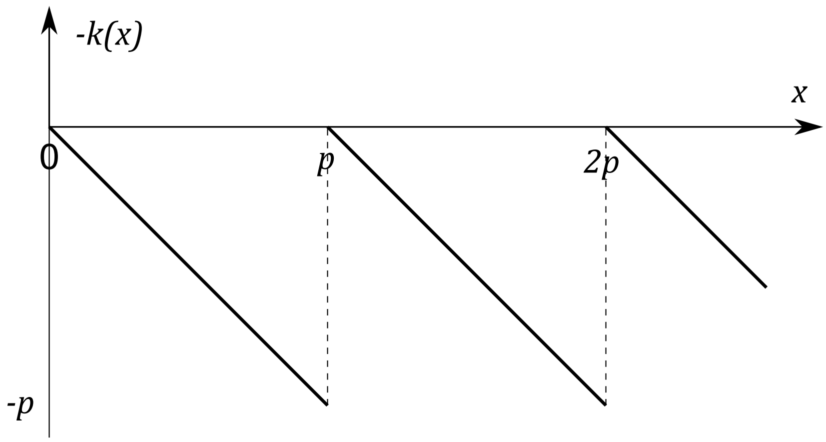
To construct a formula for $h(x)$ we take first the function

$$k(x) = x - \left\lfloor \frac{x}{p} \right\rfloor p \tag{6}$$

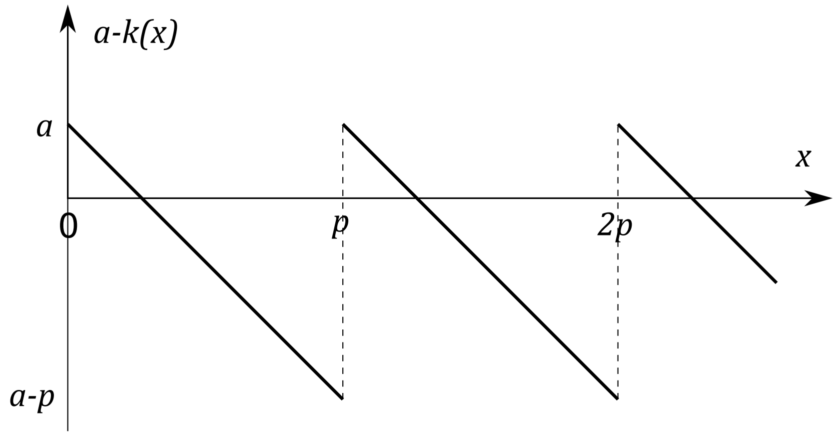
which looks as follows:



This is a *sawtooth function* (or *wave*). A little elementary work shows that this sawtooth function indeed has formula (6). Next we negate $k(x)$ in order to have $-k(x)$:



Then we add a to have $a - k(x)$:



And comparing this with the picture of $h(x)$ we see that $h(x)$ has the formula

$$h(x) = \max(\,0\,,\, a - k(x)\,). \tag{7}$$

We conclude that

$$f(x) = g(x) - \max(\,0\,,\, a - k(x)\,) \tag{8}$$

where

$$\begin{cases} g(x) = \left(\left\lfloor \dfrac{x}{p} \right\rfloor + 1\right) a, \\ k(x) = x - \left\lfloor \dfrac{x}{p} \right\rfloor p. \end{cases} \tag{9}$$

So far we have ignored points $x = np$ (n integer), but a little more careful work shows that the formulas are right for those points too. Or one can just check that the formulas work right at those points, too. Anyway, all seems to work ok with precisely these formulas.

Finally, (3) gives a solution to the original window problem as $f(x+w) - f(x)$ where x is the position of the left edge of the window.